Quantum games with correlated noise

2006 J. Phys. A: Math. Gen. 39 9321

(http://iopscience.iop.org/0305-4470/39/29/022)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 9321–9328 doi:10.1088/0305-4470/39/29/022

Quantum games with correlated noise

Ahmad Nawaz and A H Toor

Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan

E-mail: ahmad@ele.qau.edu.pk and ahtoor@qau.edu.pk

Received 5 April 2006, in final form 25 May 2006
Published 5 July 2006
Online at stacks.iop.org/JPhysA/39/9321

Abstract

We analyse quantum games with correlated noise through a generalized quantization scheme. Four different combinations on the basis of entanglement of initial quantum state and the measurement basis are analysed. It is shown that the quantum player only enjoys an advantage over the classical player when both the initial quantum state and the measurement basis are in entangled form. Furthermore, it is shown that for maximum correlation the effects of decoherence diminish and it behaves as a noiseless game.

PACS numbers: 03.67.−a, 03.65.Bz

1. Introduction

It requires the exchange of qubits between the arbiter and players to play quantum games. The transmission of a qubit through a channel is generally prone to decoherence due to its interaction with the environment. In the game theoretic sense this situation can be imagined as though there is a demon present between the arbiter and the players who corrupt the qubits. The players are not necessarily aware of the actions of the demon [1]. This type of protocol was first applied to quantum games to show that above a certain level of decoherence the quantum player has no advantage over a classical player [2]. Later, a quantum version of Prisoners' Dilemma was analysed in the presence of decoherence to prove that Nash equilibrium is not affected by decoherence [3]. Recently, Flitney and Abbott [4] showed for the quantum games based on dephasing quantum channel that the advantage that a quantum player enjoys over a classical player diminishes as decoherence increases and vanishes for the maximum decoherence.

In this paper, we analyse the quantum games based on the quantum correlated dephasing channel in the context of our generalized quantization scheme for non-zero sum games [5]. We identified four different combinations on the basis of initial state entanglement parameter, γ, and the measurement parameter, δ, for some quantum games. It is shown that for $\gamma = \delta = 0$ the game reduces to the classical and becomes independent of decoherence and memory effects. For the case when $\gamma \neq 0, \delta = 0$, the scheme reduces to Marinatto and Weber quantization scheme [6]. It is interesting to note that though the initial state is entangled, the quantum player has no advantage over the classical player. The same happens for the case of $\gamma = 0, \delta \neq 0$. However, for the case when $\gamma = \delta = \frac{\pi}{2}$, the scheme transforms to the Eisert's quantization scheme [7] and the quantum player always remains better off against a player restricted to classical strategies. Furthermore, in the limit of maximum correlation the effect of decoherence vanishes and the quantum game behaves as a noiseless game. Some interesting readings on quantum games can also be found in [8].

The paper is organized as follows: section 2 deals with the quantization of quantum games in the presence of correlated noise and a brief introduction to some classical games of interest is given in the appendix.

2. Quantization in the presence of correlated noise

Decoherence is a non-unitary dynamics that results due to the coupling of principal system with the environment. One of the important type of decoherence is phase damping or dephasing. It is uniquely quantum mechanical and describes the loss of quantum information without loss of energy. The energy eigenstate of the system does not change as a function of time during this process but the system accumulates a phase proportional to the eigenvalue. With the passage of time the relative phase between the energy eingenstates may lose.

In a pure dephasing process, a qubit transforms as

$$c\,|0\rangle + b\,|1\rangle \to c\,|0\rangle + b\,\mathrm{e}^{\mathrm{i}\phi}|1\rangle \tag{1}$$

where ϕ is the phase kick. If this phase kick, ϕ is assumed to be a random variable with Gaussian distribution of mean zero and variance 2λ then the density matrix of the system after averaging over all the values of ϕ is [11]

$$\begin{bmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{bmatrix} \to \begin{bmatrix} |a|^2 & ab^*\,\mathrm{e}^{-\lambda} \\ a^*b\,\mathrm{e}^{-\lambda} & |b|^2 \end{bmatrix}. \tag{2}$$

It is evident from the above equation that in this process the phase kicks cause the off-diagonal elements of the density matrix to decay exponentially to zero with time. In the operator sum representation, the dephasing process can be expressed as [10, 11]

$$\rho_f = \sum_{i=0}^{1} A_i \rho_{\mathrm{in}} A_i^{\dagger} \tag{3}$$

where

$$A_0 = \sqrt{1 - \frac{p}{2}}\,I, \qquad A_1 = \sqrt{\frac{p}{2}}\,\sigma_z \tag{4}$$

are the Kraus operators, I is the identity operator and σ_z is the Pauli matrix. Recognizing $1 - p = \mathrm{e}^{-\lambda}$, let N qubits be allowed to pass through such a channel then equation (3) becomes

$$\rho_f = \sum_{k_1,\ldots,k_n=0}^{N} \left(A_{k_n} \otimes \cdots A_{k_1}\right) \rho_{\mathrm{in}} \left(A_{k_1}^{\dagger} \otimes \cdots A_{k_n}^{\dagger}\right). \tag{5}$$

Now if noise is correlated with the memory of degree μ, which varies from 0 to 1 and gives the correlation strength of the quantum channel, then the Kraus operator for two-qubit system becomes [12]

$$A_{i,j} = \sqrt{p_i[(1-\mu)p_j + \mu\delta_{ij}]}\,\sigma_i \otimes \sigma_j \tag{6}$$

where $i, j = 0$ and z with $\sigma_0 = I$. Physically, this expression means that with the probability $1 - \mu$ the noise is uncorrelated and can be completely specified by the Kraus operators

$A^u_{i,j} = \sqrt{p_i p_j}\sigma_i \otimes \sigma_j$, whereas with the probability μ the noise is correlated and is specified by Kraus operators of the form $A^c_{ii} = \sqrt{p_i}\sigma_i \otimes \sigma_j$.

The protocol for quantum games in the presence of decoherence is developed in [4]. An initial entangled state is prepared by the arbiter and passed on to the players through a dephasing quantum channel. On receiving the quantum state, players apply their local operators (strategies) and return it back to arbiter through dephasing quantum channel. Then, arbiter performs the measurement and announces their payoffs.

Let the game start with the initial quantum state:

$$|\psi_{\text{in}}\rangle = \cos\frac{\gamma}{2}|00\rangle + \mathrm{i}\sin\frac{\gamma}{2}|11\rangle. \tag{7}$$

The strategies of the players in the generalized quantization scheme are represented by the unitary operator U_i of the form [5]

$$U_i = \cos\frac{\theta_i}{2}R_i + \sin\frac{\theta_i}{2}P_i, \tag{8}$$

where $i = 1$ or 2 and R_i, P_i are the unitary operators defined as

$$\begin{aligned} R_i|0\rangle &= \mathrm{e}^{\mathrm{i}\alpha_i}|0\rangle, & R_i|1\rangle &= \mathrm{e}^{-\mathrm{i}\alpha_i}|1\rangle,\\ P_i|0\rangle &= \mathrm{e}^{\mathrm{i}(\frac{\pi}{2}-\beta_i)}|1\rangle, & P_i|1\rangle &= \mathrm{e}^{\mathrm{i}(\frac{\pi}{2}+\beta_i)}|0\rangle, \end{aligned} \tag{9}$$

where $0 \leqslant \theta \leqslant \pi$, $-\pi \leqslant \alpha, \beta \leqslant \pi$. Here, we extend our earlier generalized quantization scheme to three-parameter strategy set in accordance with [4]. After the application of these strategies, the initial state given by equation (7) transforms to

$$\rho_f = (U_1 \otimes U_2)\rho_{\text{in}}(U_1 \otimes U_2)^\dagger, \tag{10}$$

where $\rho_{\text{in}} = |\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|$ is the density matrix for the quantum state. The operators used by the arbiter to determine the payoff for Alice and Bob are

$$P = \$_{00}P_{00} + \$_{01}P_{01} + \$_{10}P_{10} + \$_{11}P_{11}, \tag{11}$$

where

$$\begin{aligned} P_{00} &= |\psi_{00}\rangle\langle\psi_{00}|, & |\psi_{00}\rangle &= \cos(\delta/2)|00\rangle + \mathrm{i}\sin(\delta/2)|11\rangle, && (12a)\\ P_{11} &= |\psi_{11}\rangle\langle\psi_{11}|, & |\psi_{11}\rangle &= \cos(\delta/2)|11\rangle + \mathrm{i}\sin(\delta/2)|00\rangle, && (12b)\\ P_{10} &= |\psi_{10}\rangle\langle\psi_{10}|, & |\psi_{10}\rangle &= \cos(\delta/2)|10\rangle - \mathrm{i}\sin(\delta/2)|01\rangle, && (12c)\\ P_{01} &= |\psi_{01}\rangle\langle\psi_{01}|, & |\psi_{01}\rangle &= \cos(\delta/2)|01\rangle - \mathrm{i}\sin(\delta/2)|10\rangle, && (12d) \end{aligned}$$

with $\delta \in \left[0, \frac{\pi}{2}\right]$ and $\$_{ij}$ are the elements of payoff matrix in the ith row and jth column (given in the appendix for different games). Above payoff operators reduce to that of Eisert's scheme for δ equal to γ, which represents the entanglement of the initial state [7]. And for $\delta = 0$ the above operators transform into that of Marinatto and Weber's scheme [6]. In our extended generalized quantization to three set of parameters scheme, payoffs for the players are

$$\$^A(\theta_i, \alpha_i, \beta_i) = \mathrm{Tr}(P_A\rho_f), \qquad \$^B(\theta_i, \alpha_i, \beta_i) = \mathrm{Tr}(P_B\rho_f), \tag{13}$$

where Tr represents the trace of a matrix. Using equations (6), (7), (11) and (13), the payoffs come out to be

$$\begin{aligned} \$(\theta_i, \alpha_i, \beta_i) = c_1c_2&\left[\eta\$_{00} + \chi\$_{11} + (\$_{00} - \$_{11})\mu_p^{(1)}\mu_p^{(2)}\xi\cos 2(\alpha_1 + \alpha_2)\right]\\ &+ s_1s_2\left[\eta\$_{11} + \chi\$_{00} - (\$_{00} - \$_{11})\mu_p^{(1)}\mu_p^{(2)}\xi\cos 2(\beta_1 + \beta_2)\right]\\ &+ c_1s_2\left[\eta\$_{01} + \chi\$_{10} + (\$_{01} - \$_{10})\mu_p^{(1)}\mu_p^{(2)}\xi\cos 2(\alpha_1 - \beta_2)\right]\\ &+ c_2s_1\left[\eta\$_{10} + \chi\$_{01} - (\$_{01} - \$_{10})\mu_p^{(1)}\mu_p^{(2)}\xi\cos 2(\alpha_2 - \beta_1)\right] \end{aligned}$$

$$
\begin{aligned}
&+\frac{\mu_p^{(2)}(\$_{00}-\$_{11})}{4}\sin\theta_1\sin\theta_2\sin\delta\sin(\alpha_1+\alpha_2+\beta_1+\beta_2)\\
&+\frac{\mu_p^{(2)}(\$_{10}-\$_{01})}{4}\sin\theta_1\sin\theta_2\sin\delta\sin(\alpha_1-\alpha_2+\beta_1-\beta_2)\\
&+\frac{\mu_p^{(1)}(-\$_{00}-\$_{11}+\$_{01}+\$_{10})}{4}\sin\theta_1\sin\theta_2\sin\gamma\sin(\alpha_1+\alpha_2-\beta_1-\beta_2)
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
&\eta=\cos^2(\delta/2)\cos^2(\gamma/2)+\sin^2(\delta/2)\sin^2(\gamma/2),\\
&\chi=\cos^2(\delta/2)\sin^2\frac{\gamma}{2}+\sin^2(\delta/2)\cos^2(\gamma/2),\\
&\xi=1/2(\sin\delta\sin\gamma),\qquad c_i=\cos^2\frac{\theta_i}{2},\\
&s_i=\sin^2\frac{\theta_i}{2},\qquad \mu_p^{(i)}=(1-\mu_i)(1-p_i)^2+\mu_i.
\end{aligned}
$$

The payoff for the two players can be found by substituting the appropriate values for $\$_{ij}$ (elements of the payoff matrix for the corresponding game) into equation (14). These payoffs become the classical payoffs for $\delta=\gamma=0$ and for $\delta=\gamma=\frac{\pi}{2}$ and $\mu=0$ these payoffs transform to the results given in [4]. It is known that decoherence has no effect on the Nash equilibrium of the game but it causes a reduction in the payoffs [3, 4]. In our case, it is interesting to note that this reduction of the payoffs depends on the degree of memory μ. As μ increases from zero to one, the effect of noise reduces until finally for $\mu=1$ the payoffs become as that for noiseless game irrespective of any value of p_i. It is further noted that in comparison to memoryless case [4] the quantum phases α_i, β_i do not vanish even for the maximum value of decoherence, i.e., for $p_1=p_2=1$. To further study the effects of memory in quantum games, we consider a situation in which Alice is restricted to play classical strategies, i.e., $\alpha_1=\beta_1=0$, whereas Bob is allowed to play the quantum strategies as well. Under these circumstances following four cases for the different combinations of δ and γ are worth noting:

Case (i). When $\delta=\gamma=0$ then it is clear from equation (14) payoffs are the same as in the case of classical game [9]. These payoffs, as expected, are independent of the dephasing probabilities p_i, the quantum strategies α_2, β_2 and the memory.

Case (ii). When $\delta=0,\gamma\neq 0$ then $\eta=\cos^2\frac{\gamma}{2}$, $\chi=\sin^2\frac{\gamma}{2}$ and $\xi=0$. Using payoff matrix for the game of Prisoners Dilemma, given in the appendix, and equation (14), the payoffs for the two players are

$$
\begin{aligned}
\$^A(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(3-2\sin^2\frac{\gamma}{2}\right)+s_1s_2\left(1+2\sin^2\frac{\gamma}{2}\right)+5c_1s_2\sin^2\frac{\gamma}{2}\\
&\quad+5c_2s_1\left(1-\sin^2\frac{\gamma}{2}\right)+\frac{\mu_p^{(1)}}{4}\sin\theta_1\sin\theta_2\sin\gamma\sin(\alpha_2-\beta_2)\\
\$^B(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(3-2\sin^2\frac{\gamma}{2}\right)+s_1s_2\left(1+2\sin^2\frac{\gamma}{2}\right)+5c_1s_2\left(1-\sin^2\frac{\gamma}{2}\right)\\
&\quad+5c_2s_1\sin^2\frac{\gamma}{2}+\frac{\mu_p^{(1)}}{4}\sin\theta_1\sin\theta_2\sin\gamma\sin(\alpha_2-\beta_2).
\end{aligned}
\tag{15}
$$

In this case, the optimal strategy for the quantum player, Bob, is $\alpha_2-\beta_2=\frac{\pi}{2}$. Though his choice for θ_2 depends on Alice's choice for θ_1, but he can play $\theta_2=\frac{\pi}{2}$, without being bothered about Alice's choice as rational reasoning leads Alice to play $\theta_1=\frac{\pi}{2}$. Under these choices of moves the payoffs for the two players are equal:

$$\$^A\left(\frac{\pi}{2},\frac{\pi}{2},\alpha_2-\beta_2=\frac{\pi}{2}\right)=\$^B\left(\frac{\pi}{2},\frac{\pi}{2},\alpha_2-\beta_2=\frac{\pi}{2}\right)=\frac{9}{4}+\frac{\mu_p^{(1)}}{4}\sin\gamma. \tag{16}$$

It is evident that the quantum player has no advantage over the classical player. Similarly, for the Chicken game the payoffs for the two players are

$$\begin{aligned}\$^A(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(3-3\sin^2\frac{\gamma}{2}\right)+s_1s_2\left(3\sin^2\frac{\gamma}{2}\right)+c_1s_2\left(3\sin^2\frac{\gamma}{2}+1\right)\\&\quad+c_2s_1\left(4-3\sin^2\frac{\gamma}{2}\right)+\frac{\mu_p^{(1)}}{2}\sin\theta_1\sin\theta_2\sin\gamma\sin(\alpha_2-\beta_2) \qquad (17)\\\$^B(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(3-3\sin^2\frac{\gamma}{2}\right)+s_1s_2\left(3\sin^2\frac{\gamma}{2}\right)+c_1s_2\left(4-3\sin^2\frac{\gamma}{2}\right)\\&\quad+c_2s_1\left(1+3\sin^2\frac{\gamma}{2}\right)+\frac{\mu_p^{(1)}}{2}\sin\theta_1\sin\theta_2\sin\gamma\sin(\alpha_2-\beta_2) \qquad (18)\end{aligned}$$

and it can be shown using the same argument as for the game of Prisoner Dilemma that the quantum player does not have any advantage over classical player in the Chicken game as well.

For the case of the quantum Battle of Sexes, the payoffs become

$$\begin{aligned}\$^A(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(2-\sin^2\frac{\gamma}{2}\right)+s_1s_2\left(1+\sin^2\frac{\gamma}{2}\right)\\&\quad-\frac{3\mu_p^{(1)}}{4}\sin\theta_1\sin\theta_2\sin\gamma\sin(\alpha_2-\beta_2)\\\$^B(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(1+\sin^2\frac{\gamma}{2}\right)+s_1s_2\left(2-\sin^2\frac{\gamma}{2}\right)\\&\quad-\frac{3\mu_p^{(1)}}{4}\sin\theta_1\sin\theta_2\sin\gamma\sin(\alpha_2-\beta_2). \qquad (19)\end{aligned}$$

Here, the optimal strategy for Bob is $\alpha_2-\beta_2=-\frac{\pi}{2}$ and $\theta_2=\frac{\pi}{2}$, keeping in view that the best strategy for Alice is $\theta_1=\frac{\pi}{2}$. The corresponding payoffs of the players are again equal for these choices, i.e.,

$$\$^A\left(\frac{\pi}{2},\frac{\pi}{2},\alpha_2-\beta_2=-\frac{\pi}{2}\right)=\$^B\left(\frac{\pi}{2},\frac{\pi}{2},\alpha_2-\beta_2=-\frac{\pi}{2}\right)=\frac{3}{4}+\frac{3}{4}\mu_p^{(1)}\sin\gamma. \tag{20}$$

It is clear that for the case $\delta=0,\gamma\neq0$ the quantum player has no advantage over the classical player for three games considered above. It is interesting because the game starts from an entangled state and the payoffs are also the functions of the quantum phases, α_i,β_i, dephasing probability, p_1, and the degree of memory, μ_1, of the quantum channel between Bob and arbiter.

Case (iii). When $\delta\neq0,\gamma=0$ then using equation (14) the payoffs for the two players in games of Prisoners Dilemma, Chicken and Battle of Sexes are

$$\begin{aligned}\$^A(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(3-2\sin^2\frac{\delta}{2}\right)+s_1s_2\left(1+2\sin^2\frac{\delta}{2}\right)\\&\quad+\frac{7\mu_p^{(2)}}{4}\sin\theta_1\sin\theta_2\sin\delta\sin(\alpha_2+\beta_2)\\\$^B(\theta_1,\theta_2,\alpha_2,\beta_2)&=c_1c_2\left(1+\sin^2\frac{\delta}{2}\right)+s_1s_2\left(2-\sin^2\frac{\delta}{2}\right)\\&\quad-\frac{3\mu_p^{(2)}}{4}\sin\theta_1\sin\theta_2\sin\delta\sin(\alpha_2+\beta_2), \qquad (21)\end{aligned}$$

$$
\begin{aligned}
\$^{A}(\theta_1,\theta_2) &= c_1c_2\left(3-3\sin^2\frac{\delta}{2}\right)+s_1s_2\left(3\sin^2\frac{\delta}{2}\right)\\
&\quad +c_1s_2\left(1+3\sin^2\frac{\delta}{2}\right)+c_2s_1\left(4-3\sin^2\frac{\delta}{2}\right)\\
\$^{B}(\theta_1,\theta_2,\alpha_2,\beta_2) &= c_1c_2\left(3-3\sin^2\frac{\delta}{2}\right)+s_1s_2\left(3\sin^2\frac{\delta}{2}\right)+c_1s_2\left(4-3\sin^2\frac{\delta}{2}\right)\\
&\quad +c_2s_1\left(1+3\sin^2\frac{\delta}{2}\right)+\frac{3\mu_p^{(2)}}{2}\sin\theta_1\sin\theta_2\sin\delta\sin(\alpha_2+\beta_2),
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\$^{A}(\theta_1,\theta_2,\alpha_2,\beta_2) &= c_1c_2\left(2-\sin^2\frac{\delta}{2}\right)+s_1s_2\left(1+\sin^2\frac{\delta}{2}\right)\\
&\quad +\frac{3\mu_p^{(2)}}{4}\sin\theta_1\sin\theta_2\sin\delta\sin(\alpha_2+\beta_2)\\
\$^{B}(\theta_1,\theta_2,\alpha_2,\beta_2) &= c_1c_2\left(1+\sin^2\frac{\delta}{2}\right)+s_1s_2\left(2-\sin^2\frac{\delta}{2}\right)\\
&\quad -\frac{3\mu_p^{(2)}}{4}\sin\theta_1\sin\theta_2\sin\delta\sin(\alpha_2+\beta_2),
\end{aligned}
\tag{23}
$$

respectively. It is evident from the above expressions for the payoffs that the optimal strategy for Bob, the quantum player, is $\alpha_2+\beta_2=-\frac{\pi}{2}$, with $\theta_2=\frac{\pi}{2}$, for Prisoners Dilemma and Battle of Sexes. But corresponding payoff for Alice is less. However, she can overcome this by playing $\theta_1=0$ or π, so that the payoffs for both the players become independent of the quantum phases α_2,β_2. So there remains no option for the quantum player to enhance his payoff by exploiting the quantum move. However, in the case of Chicken game the quantum player can enhance his payoff without affecting the payoff of the classical player. But again the classical player has the ability to prevent quantum strategies by playing $\theta_1=0$ or π. So, there remains no advantage for playing quantum strategies. It is also interesting to note that though by playing this move Alice could force the payoffs of the two players to be independent of dephasing factor p_2 and the degree of memory μ_2, however, the game remains different from its classical counterpart.

Case (*iv*). When $\delta=\gamma=\frac{\pi}{2}$ then equation (14) with $\mu_1=\mu_2=0$ gives the same results as mentioned in [4] and the quantum player is better off for $p<1$. However, when decoherence increases this advantage diminishes and vanishes for maximum decoherence, i.e., $p=1$. But in our case when $\mu\neq 0$, the quantum player is always better off even for maximum noise, i.e., $p=1$, which was not possible in memoryless case. Furthermore, it is worth noting that as the degree of memory increases from 0 to 1 the effect of noise on the payoffs starts decreasing and for $\mu=1$ it behaves like a noiseless game. In the case of Prisoners Dilemma, the optimal strategy for Bob is to play $\alpha_2=\frac{\pi}{2}$ and $\beta_2=0$. His choice for θ_2 is $\frac{\pi}{2}$, independent of Alice's move. The payoffs for Alice and Bob as a function of decoherence probability $p_1=p_2=p$ at $\mu=\frac{1}{2}$ are

$$
\begin{aligned}
\$^{A}(\theta_1,\theta_2,\alpha_2,\beta_2) &= c_1c_2\left[2+\mu_p^2\cos 2\alpha_2\right]+s_1s_2\left[2-\mu_p^2\cos 2\beta_2\right]\\
&\quad +\frac{5}{2}c_1s_2\left[1-\mu_p^2\cos 2\beta_2\right]+\frac{5}{2}c_2s_1\left[1+\mu_p^2\cos 2\alpha_2\right]\\
&\quad +\frac{\mu_p}{4}\sin\theta_1\sin\theta_2\sin(\alpha_2-\beta_2)-\frac{3\mu_p}{4}\sin\theta_1\sin\theta_2\sin(\alpha_2+\beta_2)
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
\$^B(\theta_1, \theta_2, \alpha_2, \beta_2) = {} & c_1c_2\big[2 + \mu_p^2\cos 2\alpha_2\big] + s_1s_2\big[2 - \mu_p^2\cos 2\beta_2\big] \\
& + \frac{5}{2}c_1s_2\big[1 + \mu_p^2\cos 2\beta_2\big] + \frac{5}{2}c_2s_1\big[1 - \mu_p^2\cos 2\alpha_2\big] \\
& + \frac{7\mu_p}{4}\sin\theta_1\sin\theta_2\sin(\alpha_2 + \beta_2) + \frac{\mu_p}{4}\sin\theta_1\sin\theta_2\sin(\alpha_2 - \beta_2) \qquad (25)
\end{aligned}
$$

where

$$\mu_p = \frac{1 + (1 - p)^2}{2}.$$

It is obvious from above payoffs that quantum player Bob can always out perform Alice, for all values of p. Similarly, for the case of Chicken and Battle of Sexes game, it can be proved that the classical player can be out performed by Bob, at $\alpha_2 = \frac{\pi}{2}$, $\beta_2 = 0$ and $\theta_2 = \frac{\pi}{2}$ and $\alpha_2 = -\frac{\pi}{2}$, $\beta = 0$ and $\theta_2 = \frac{\pi}{2}$, respectively.

3. Conclusion

Quantum games with correlated noise are studied under the generalized quantization scheme [5]. Three games, Prisoner Dilemma, Battle of Sexes and Chicken are studied with one player restricted to the classical strategy while the other is allowed to play quantum strategies. It is shown that the effects of the memory and decoherence become effective for the case $\gamma = \delta = \frac{\pi}{2}$, for which the quantum player out performs the classical player. It is also shown that memory controls payoffs reduction due to decoherence and for the limit of maximum memory decoherence becomes ineffective.

Appendix. Some classical games

Here, we briefly describe three classical games on interest.

Prisoner's Dilemma

This game depicts a situation where two suspects (players), who have committed a crime together, are being interrogated in a separate cell. The two possible moves for each player are to cooperate (C), i.e., not to confess the crime or to defect (D), i.e., to confess the crime without any communication between them but having access to the following payoff matrix:

$$
\begin{array}{cc}
 & \text{Bob} \\
 & \begin{array}{cc} C & D \end{array} \\
\text{Alice} \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} (3,3) & (0,5) \\ (5,0) & (1,1) \end{bmatrix}.
\end{array} \qquad (A.1)
$$

It is obvious from the payoff matrix (A.1) that D is the dominant strategy for the two players. Therefore, rational reasoning forces the players to play D. Thus, (D, D) is the Nash equilibrium of this game with payoffs $(1, 1)$. But the players could get higher payoffs if they would have played C instead of D. This is the dilemma in this game.

The Chicken game

The payoff matrix for this game is

$$
\begin{array}{cc}
 & \text{Bob} \\
 & \begin{array}{cc} C & D \end{array} \\
\text{Alice} \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} (3,3) & (1,4) \\ (4,1) & (0,0) \end{bmatrix}.
\end{array} \qquad (A.2)
$$

In this game two players drove their cars towards each other. The first one to swerve to avoid collision is the loser (Chicken) and the one who keeps on driving straight is the winner. The two possible strategies for each player are C (Cooperate) to swerve and D (Defect) not to swerve. There is no dominant strategy in this game. There are two Nash equilibria (C, D) and (D, C), the former is preferred by Bob and the latter is preferred by Alice. The dilemma of this game is that the Pareto optimal strategy (C, C) is not Nash equilibrium.

Battle of Sexes
The payoff matrix for this game is

$$\begin{array}{cc} & \text{Bob} \\ & \begin{array}{cc} O & T \end{array} \\ \text{Alice} \begin{array}{c} O \\ T \end{array} & \begin{bmatrix} (2,1) & (0,0) \\ (0,0) & (1,2) \end{bmatrix} \end{array}. \tag{A.3}$$

In the usual exposition of this game Alice is fond of Opera whereas Bob likes watching TV but they also want to spend the evening together. In the absence of communication they face a dilemma in choosing their strategies. In the game matrix (A.3) O and T represent Opera and TV, respectively.

References

[1] Lee C F and Johnson N F 2002 *Physics World* October
[2] Johnson N F 2001 *Phys. Rev.* A **63** 020302
[3] Chen L K, Ang H, Kiang D, Kwek L C and Lo C F 2003 *Phys. Lett.* A **316** 317
[4] Flitney A P and Abbott D 2005 *J. Phys. A: Math. Gen.* **38** 449
[5] Nawaz A and Toor A H 2004 *J. Phys. A: Math. Gen.* **37** 11457 (*Preprint* quant-ph/0409046)
[6] Marinatto L and Weber T 2000 *Phys. Lett.* A **272** 291 (*Preprint* quant-ph/0004081)
[7] Eisert J, Wilkens M and Lewenstein M 1999 *Phys. Rev. Lett.* **83** 3077
[8] Arfi B 2005 *Theory Decis.* **59** 127
Lo C F and Kiang D 2005 *Phys. Lett.* A **346** 65
Qin G, Chen X, Sun M and Du J F 2005 *J. Phys. A: Math. Gen.* **38** 4247
Piotrowski E W and Sladkowski J 2004 *Quant. Finance* **4** C61
Peterson I 1999 *Sci. News* **156** 334
Chao A 2002 *New Sci.* **173** 12
[9] Eisert J and Wilkens M 2000 *J. Mod. Opt.* **47** 2543 (*Preprint* quant-ph/0004076)
[10] Kraus K 1983 *State, Effects and Operations: Fundamental Notions of Quantum Theory (Lecture Notes in Physics* vol 190*)* (Berlin: Springer)
[11] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[12] Macchiavello C and Palma G M 2002 *Phys. Rev.* A **65** 050301